B. Math. I Year
First Semester 2000 - 2001
Back Paper / Analysis

1. a) Find $\lim\limits_{n\to\infty} \frac{2n^{3/2}+5}{5n-17}$.

   b) Find $\lim\limits_{n\to\infty} \left(1+\frac{1}{n}\right)^{n^2}$. [5+5]

2. If $a_n \geq 0$ and $\sum\limits_{n=1}^{\infty} a_n$ is convergent, prove that $\sum\limits_{n=1}^{\infty} \frac{a_n^2}{a_n+1}$ is convergent. [5]

3. Prove that $\forall\, x \in \boldsymbol{R}$, $\sum\limits_{n=1}^{\infty} \frac{x^n}{(n!)^2}$ is absolutely convergent. [5]

4. Prove that $f(x) = \frac{1}{x}$ is not uniformly continuous on $(0,1]$. [10]

5. The function $f(x) = x^2$ is Riemann integrable on $[0,1]$. Justify this by quoting an appropriate theorem. Find $\int\limits_0^1 x^2 dx$ from *first principles.* [10]

6. Find the third Taylor polynomial of $f(x) = e^x \sin x$ around $a = 0$. Use this to evaluate $f(0.001)$. Estimate the maximum error involved in this method. [20]

7. If $f(x) = \int\limits_x^{x+1} g(t)dt$. Find $f'(x)$. [10]

8. If $f \geq 0$ on $[0,1]$ and continuous. Prove that $\int\limits_0^1 f(t)dt = 0$ iff $f \equiv 0$. [10]

9. Give an example of $f \geq 0$ on $[0,\infty)$, such that f is *unbounded* but $\int\limits_0^{\infty} f(t)dt$ is finite. [10]

10. If f_n is a sequence of continuous functions on $[0,1]$, such that $f_n(x) \to 0$ for each x, does it follow $\int\limits_0^1 f_n(x)dx \to 0$. If true, prove it. If false, give a counter example. [10]